## CHAPTER <br> PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

## Syllabus

> Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency.
$>$ Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically: by substitution, by elimination and by cross-multiplication method. Simple situational problems. Simple problems on equations reducible to linear equations.

| List of Concepts | 2018 |  | 2019 |  | 2020 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Delhi | Outside Delhi | Delhi | Outside Delhi | Delhi | Outside Delhi |
| Graphical Solution of Linear equations in two variables and relation with consistency or inconsistency |  |  | 2 Q (2 M) | 2 Q (2 M) | $\begin{aligned} & 1 \mathrm{Q}(1 \mathrm{M}) \\ & 1 \mathrm{Q}(3 \mathrm{M}) \end{aligned}$ | 1 Q (1 M) |
| Algebraic methods to solve pair of Linear equations | 1 Q (2 M) |  | $\begin{aligned} & 2 \mathrm{Q}(3 \mathrm{M}) \\ & 1 \mathrm{Q}(4 \mathrm{M}) \end{aligned}$ | 2 Q (2 M) | 1 Q (3 M) | $\begin{aligned} & 1 \text { Q (3 M) } \\ & 1 Q(4 \mathrm{M}) \end{aligned}$ |

## TOPIC - 1

## Graphical Solution of Linear Equations in Two Variables

## E Revision Notes

$>$ Linear equation in two variables: An equation in the form of $a x+b y+c=$ 0 , where $a, b$ and $c$ are real numbers and $a$ and $b$ are not zero, is called a linear equation in two variables $x$ and $y$.
General form of a pair of linear equations in two variables is:

$$
\begin{array}{ll}
\text { and } \quad & \begin{array}{l}
a_{1} x+b_{1} y+c_{1}=0 \\
a_{2} x+b_{2} y+c_{2}=0,
\end{array}
\end{array}
$$

where $a_{1}, a_{2}, b_{1}, b_{2}, c_{1}$ and $c_{2}$ are real numbers, such that

$$
\begin{array}{lr}
\text { e.g., } & 3 x-y+7=0, \\
\text { and } & 7 x+y=3
\end{array}
$$

are linear equations in two variables $x$ and $y$.
> There are two methods of solving simultaneous linear equations in two variables:
(i) Graphical method, and
(ii) Algebraic methods.

1. Graphical Method:
(i) Express one variable (say $y$ ) in terms of the other variable $x, y=a x+b$, for the given equation.
(ii) Take three values of independent variable $x$ and find the corresponding values of dependent variable $y$, take integral values only.
(iii) Plot these values on the graph paper in order to represent these equations.
(iv) If the lines intersect at a distinct point, then point of intersection will be the unique solution for given equations. In this case, the pair of linear equations is consistent.
If $\frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}}$, then the pair of linear equations is consistent with a unique solution.


Intersecting Lines
(v) If the lines representing the linear equations coincides, then system of equations has infinitely many solutions. In this case, the pair of linear equations is consistent and dependent.
If $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}$, then the pair of linear equations is consistent with infinitely many solutions.


## Coincident Lines

(vi) If the lines representing the pair of linear equations are parallel, then the system of equations has no solution and is called inconsistent.
If $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}}$, then the pair of linear equations is inconsistent with no solution.


Parallel Lines

## Possibilities of solutions and Inconsistency:

$\left.\begin{array}{|c|c|c|c|c|c|c|c|}\hline \text { Pair of lines } & \frac{a_{1}}{a_{2}} & \frac{b_{1}}{b_{2}} & \frac{c_{1}}{c_{2}} & \begin{array}{c}\text { Compare the } \\ \text { ratios }\end{array} & \begin{array}{c}\text { Graphical } \\ \text { representation }\end{array} & \begin{array}{c}\text { Algebraic } \\ \text { interpretation }\end{array} & \begin{array}{c}\text { Conditions for } \\ \text { solvability }\end{array} \\ \hline \begin{array}{r}x-4 y-2 y=0\end{array} & \frac{1}{3} & \frac{-2}{-4} & \frac{0}{-20} & \frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}} & \begin{array}{c}\text { Intersecting } \\ \text { lines }\end{array} & \begin{array}{c}\text { Exactly one } \\ \text { solution or } \\ \text { Unique solution }\end{array} & \begin{array}{c}\text { System is } \\ \text { consistent }\end{array} \\ \hline \begin{array}{r}2 x+3 y-9=0 \\ 4 x+6 y-18=0\end{array} & \frac{2}{4} & \frac{3}{6} & \frac{-9}{-18} & \frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}} & \begin{array}{c}\text { Coincident } \\ \text { lines }\end{array} & \begin{array}{c}\text { Infinitely many } \\ \text { solutions }\end{array} & \begin{array}{c}\text { System is } \\ \text { consistent }\end{array} \\ \hline x+2 y-4=0 & \frac{1}{2} & \frac{2}{4} & \frac{-4}{-12} & \frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}} & \text { Parallel } & \text { lines } & \text { No solution }\end{array} \begin{array}{c}\text { System is } \\ \text { inconsistent }\end{array}\right]$

## Know the Formulae

If $a_{1} x+b_{1} y+c_{1}=0$ and $a_{2} x+b_{2} y+c_{2}=0$ is a pair of linear equations in two variables $x$ and $y$ such that:
(i) System has unique solution
if

$$
\frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}}
$$

(ii) System has infinite number of solutions

$$
\text { if } \begin{aligned}
\frac{a_{1}}{a_{2}} & =\frac{b_{1}}{b_{2}} \\
& =\frac{c_{1}}{c_{2}}
\end{aligned}
$$

(iii) System has no solution
if

$$
\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}}
$$

## Mnemonics

Algebra Methods

1. Concept:

System has unique solution

$$
\frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}}
$$

## For unique feature Audi $\mathbf{A}_{1}$ and $\mathbf{A}_{2}$ are not same as $B M W \mathbf{B}_{1}$ and $\mathbf{B}_{\mathbf{2}}$

Interpretation:

$$
\begin{aligned}
& \mathrm{A}_{1}=a_{1} \\
& \mathrm{~A}_{2}=a_{2} \\
& \mathrm{~B}_{1}=b_{1} \\
& \mathrm{~B}_{2}=b_{2}
\end{aligned}
$$

## How is it done on the GREENBOARD?

Q.l. From the given pair of linear equations

$$
3 x+a y=50
$$

and $\quad 9 x-21 y=15$,
find the value of a for them to be parallel
Solution
Step I: Given $3 x+a y=50$ and

$$
9 x-21 y=15
$$

$\Rightarrow \quad a_{1}=3, b_{1}=a, c_{1}=-50$
and $a_{2}=9, b_{2}=-21, c_{2}=-15$
Step II: For lines to be parallel

$$
\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}}
$$

Step III:

$$
\begin{aligned}
\frac{3}{9} & =\frac{a}{-21} \neq \frac{-50_{1}}{-15} \\
a & =\frac{1}{3} \times(-21) \\
a & =-7
\end{aligned}
$$

## Very Short Answer Type Questions

## 1 mark each

Q.1. For what value of $k$, the pair of linear equations $3 x+y=3$ and $6 x+k y=8$ does not have a solution.

A [CBSE SQP, 2020-21]
Sol. System has no solution if

$$
\begin{align*}
\frac{a_{1}}{a_{2}} & =\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}} \\
\frac{3}{6} & =\frac{1}{k} \neq \frac{3}{8} \\
\frac{3}{6} & =\frac{1}{k} \\
k & =2
\end{align*}
$$

[CBSE Marking Scheme, 2020-21]
Detailed Solution:
Given equation are:

$$
\begin{equation*}
3 x+y-3=0 \tag{i}
\end{equation*}
$$

and $\quad 6 x+k y-8=0$
Comparing eq. (i) with $a_{1} x+b_{1} y+c_{1}=0$ and eq. (ii)
with $a_{2} x+b_{2} y+c_{2}=0$, we get,
$a_{1}=3, a_{2}=6, b_{1}=1, b_{2}=k, c_{1}=-3$ and $c_{2}=-8$
Since, given equations has no solution.

$$
\begin{array}{ll}
\text { So, } & \frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}} \\
\Rightarrow & \frac{3}{6}=\frac{1}{k} \neq \frac{-3}{-8} \\
\Rightarrow & \frac{1}{2}=\frac{1}{k} \neq \frac{3}{8} \\
& \text { Either } \frac{1}{2}=\frac{1}{k} \text { or } \frac{1}{k} \neq \frac{3}{8} \\
\Rightarrow & k=2 \text { or } k \neq \frac{3}{8}
\end{array}
$$

## COMMONLY MADE ERROR

- Some students used wrong condition for the solution of linear equations.


## ANSWERING TIP

- Candidates must be familiar with all three conditions for solvability like unique, infinitely many solutions and no solution.
Q. 2. For what value of $k$, the pair of linear equations $x+y-4=0$ and $2 x+k y=3$ does not have a solution.

> A + U [CBSE Delhi Set-I, 2020]

Sol. Given equations:

$$
\begin{aligned}
& \quad x+y-4=0 \\
& \text { and } \begin{aligned}
& 2 x+k y-3=0 \\
& \text { Here } \frac{a_{1}}{a_{2}}=\frac{1}{2}, \frac{b_{1}}{b_{2}}=\frac{1}{k} \text { and } \frac{c_{1}}{c_{2}}=\frac{-4}{-3}=\frac{4}{3}
\end{aligned}=\frac{4}{}
\end{aligned}
$$

$\because$ System has no solution

$$
\begin{array}{ll}
\therefore & \frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}} \\
\Rightarrow & \frac{1}{2}=\frac{1}{k} \neq \frac{4}{3} \\
\Rightarrow & k=2 \text { or } k \neq \frac{3}{4}
\end{array}
$$

Hence, the value of $k$ is 2 .
[CBSE Marking Scheme, 2020]
Q.3. Find the value of $k$ for which system of linear equations $x+2 y=3,5 x+k y+7=0$ is inconsistent
[CBSE OD Set-I, 2020]
Sol. We have,

$$
\begin{align*}
x+2 y-3 & =0  \tag{i}\\
\text { and } \quad & 5 x+k y+7 \tag{ii}
\end{align*}
$$

Comparing eq. (i) with $a_{1} x+b_{1} y+c_{1}=0$ and eq. (ii) with $a_{2} x+b_{2} y+c_{2}=0$, we get.
$a_{1}=1, a_{2}=5, b_{1}=2$ and $b_{2}=k, c_{1}=-3, c_{2}=7$
Since, system is inconsistent, then

$$
\begin{aligned}
& \frac{a_{1}}{a_{2}}
\end{aligned}=\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}}
$$

$$
\text { Either } \frac{1}{5}=\frac{2}{k} \text { or } \frac{2}{k} \neq \frac{-3}{7}
$$

$$
\Rightarrow \quad 10=k \text { or } k \neq \frac{-14}{3}
$$

Hence, the value of $k=10$.
[AI] Q.4. For which value $(s)$ of $p$, will the lines represented by the following pair of linear equations be parallel:

$$
\begin{array}{r}
3 x-y-5=0 \\
6 x-2 y-p=0
\end{array}
$$

R [CBSE SQP, 2020]
Sol. All real values except 10 .
[CBSE SQP Marking Scheme, 2020]

## Detailed Solution:

Given, $\quad 3 x-y-5=0$

$$
\begin{equation*}
6 x-2 y-p=0 \tag{i}
\end{equation*}
$$

Comparing eq. (i) with $a_{1} x+b_{1} y+c_{1}=0$ and eq. (ii) with $a_{2} x+b_{2} y+c_{2}=0$, we get, $a_{1}=3, a_{2}=6, b_{1}=-1, b_{2}=-2, c_{1}=-5$ and $c_{2}=-p^{1 / 2}$ We know that, for parallel lines

$$
\begin{aligned}
\frac{a_{1}}{a_{2}} & =\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}} \\
\frac{3}{6} & =\frac{1}{2} \neq \frac{5}{p}
\end{aligned}
$$

From (i) and (ii),

$$
p \neq 10
$$

So, $p$ can have any number other than 10 . $1 / 2$

## Short Answer Type Questions-I

## 2 marks each

(AI) Q. 1. Find $c$ if the system of equations $c x+3 y+(3-c)$ $=0 ; 12 x+c y-c=0$ has infinitely many solutions ? U +R [CBSE Delhi Set-I, 2019]
Sol. System of equations has infinitely many solutions

$$
\begin{array}{llrl}
\therefore & \frac{c}{12}=\frac{3}{c}=\frac{3-c}{-c} \\
\Rightarrow & c^{2}=36 & \Rightarrow c=6 \text { or } c=-6 \\
\text { Also } & -3 c=3 c-c^{2} \Rightarrow c=6 \text { or } c=0
\end{array}
$$

From equations (1) and (2)

$$
c=6 .
$$

[CBSE Marking Scheme, 2019]

## Detailed Solution:

The given equations are:

$$
\begin{array}{rlrl} 
& & c x+3 y+(3-c) & =0 \\
& \text { and } & 12 x+c y-c & =0 \\
& \text { Here, } \quad \frac{a_{1}}{a_{2}}=\frac{c}{12}, \frac{b_{1}}{b_{2}}=\frac{3}{c}, \frac{c_{1}}{c_{2}} & =\frac{3-c}{-c}
\end{array}
$$

For infinitely many solutions:

$$
\begin{array}{rlrl} 
& & \frac{a_{1}}{a_{2}} & =\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}} \\
& \therefore \text { For, } & \frac{a_{1}}{a_{2}} & =\frac{c_{1}}{c_{2}} \\
\Rightarrow & \frac{c}{12} & =\frac{3-c}{-c} \\
\Rightarrow & -c^{2} & =36-12 c \\
\Rightarrow & -c^{2}+12 c-36 & =0
\end{array}
$$

$$
\begin{array}{rlrl}
\Rightarrow & c^{2}-12 c+36 & =0 \\
\Rightarrow & c^{2}-6 c-6 c+36 & =0 \\
\Rightarrow & c(c-6)-6(c-6) & =0 \\
\Rightarrow & (c-6)(c-6) & =0 \\
\Rightarrow & c & =6,6 \\
& \text { and for, } & \frac{b_{1}}{b_{2}} & =\frac{c_{1}}{c_{2}} \\
\Rightarrow & \frac{3}{c} & =\frac{3-c}{-c} \\
\Rightarrow & -3 c & =3 c-c^{2} \\
\Rightarrow & c^{2}-6 c & =0 \\
\Rightarrow & c(c-6) & =0 \\
\Rightarrow & & c=6 \text { or } c & =0
\end{array}
$$

Hence, the value of $c$ is 6 , for which the given equations have infinitely many solutions.

## COMMONLY MADE ERROR

- Students often get confused between the conditions of unique, Infinite and no solution.


## ANSWERING TIP

- Understand the difference between the conditions of Unique solution, infinite solutions and no solution and apply the same carefully.
Q. 2. For what value of $k$, the following pair of linear equations have infinitely many solutions:

$$
2 x+3 y=7 \text { and }(k+1) x+(2 k-1) y=4 k+1
$$

A [CBSE Delhi Set-II, 2019]
Sol. The given equations are:

$$
2 x+3 y=7
$$

and $(k+1) x+(2 k-1) y=4 k+1$
Here $\frac{a_{1}}{a_{2}}=\frac{2}{k+1}, \frac{b_{1}}{b_{2}}=\frac{3}{2 k-1}$
and $\quad \frac{c_{1}}{c_{2}}=\frac{-7}{-(4 k+1)}$
For infinitely many solutions,

$$
\begin{align*}
& \frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}} \\
& \text { i.e., } \quad \frac{2}{k+1}=\frac{3}{2 k-1}=\frac{7}{4 k+1} \\
& \text { For } \quad \frac{2}{k+1}=\frac{3}{2 k-1} \\
& \Rightarrow \quad 2(2 k-1)=3(k+1) \\
& \Rightarrow \quad 4 k-2=3 k+3 \\
& \Rightarrow \quad 4 k-3 k=3+2 \\
& \Rightarrow \quad k=5 \\
& \text { and for, } \\
& \frac{2}{k+1}=\frac{7}{4 k+1} \\
& \Rightarrow \quad 2(4 k+1)=7(k+1) \\
& \Rightarrow \quad 8 k+2=7 k+7 \\
& \Rightarrow \quad 8 k-7 k=7-2 \\
& \Rightarrow \quad k=5 \\
& 1 / 2 \\
& \text { and for, } \\
& \frac{3}{2 k-1}=\frac{7}{4 k+1} \\
& \Rightarrow \quad 7(2 k-1)=3(4 k+1) \\
& \Rightarrow \quad 14 k-7=12 k+3 \\
& \Rightarrow \quad 14 k-12 k=3+7 \\
& \Rightarrow \quad 2 k=10 \\
& \Rightarrow \quad k=5
\end{align*}
$$

Hence, the value of $k$ is 5 , for which the given equations have infinitely many solutions.
[CBSE Marking Scheme, 2019]
[AI] Q.3. Find the value(s) of $k$ so that the pair of equations $x+2 y=5$ and $3 x+k y+15=0$ has a unique solution.

A [CBSE OD-I, 2019]
Sol. For unique solution $\frac{1}{3} \neq \frac{2}{k}$

$$
\Rightarrow \quad k \neq 6
$$

[CBSE Marking Scheme, 2019]

## Detailed Solution:

Given equations are

$$
\begin{equation*}
x+2 y-5=0 \tag{i}
\end{equation*}
$$

and $\quad 3 x+k y+15=0$
Comparing eq (i) with $a_{1} x+b_{1} y+c_{1}=0$ and eq (ii) with $a_{2} x+b_{2} y+c_{2}=0$, we get
$a_{1}=1, a_{2}=3, b_{1}=2, b_{2}=k, c_{1}=-5$ and $c_{2}=15$
Since, given equations have unique solution, So,

$$
\begin{array}{lll} 
& \frac{a_{1}}{a_{2}} & \neq \frac{b_{1}}{b_{2}} \\
\text { i.e. } & \frac{1}{3} & \neq \frac{2}{k} \\
\Rightarrow & k & \neq 6
\end{array}
$$

Hence, for all values of $k$ except 6 , the given pair of equations have unique solution.

## COMMONLY MADE ERROR

- Many students make mistakes while applying condition of unique solution. They forget to insert the 'not equals to' $\operatorname{sign}(\neq)$.


## ANSWERING TIP

- It is necessary to practice more such questions for better understanding of the condition of unique solution.
[AI Q. 4. For what value of $k$, will the following pair of equations have infinitely many solutions:
$2 x+3 y=7$ and $(k+2) x-3(1-k) y=5 k+1$
U [CBSE OD Set-2, 2019]
Sol. Given equation are:

$$
\begin{equation*}
2 x+3 y=7 \tag{i}
\end{equation*}
$$

and $(k+2) x-3(1-k) y=5 k+1$
Comparing eq. (i) with $a_{1} x+b_{1} y+c_{1}=0$ and eq.
(ii) with $a_{2} x+b_{2} y+c_{2}=0$, we get
$a_{1}=2, b_{1}=3, c_{1}=-7, a_{2}=k+2, b_{2}=-3(1-\mathrm{k})$ and $c_{2}=-(5 k+1)$
For infinitely many solutions,

$$
\begin{align*}
& \frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}} \\
& \text { i.e., } \quad \frac{2}{k+2}=\frac{3}{-3(1-k)}=\frac{-7}{-(5 k+1)} \\
& \frac{2}{k+2}=\frac{3}{-3(1-k)} \\
& \Rightarrow \quad 2 \times[-3(1-k)]=3(k+2) \\
& \Rightarrow \quad-6(1-k)=3(k+2) \\
& \Rightarrow \quad-6+6 k=3 k+6 \\
& \Rightarrow \quad 6 k-3 k=6+6 \\
& \Rightarrow \quad 3 k=12 \\
& \Rightarrow \quad k=4 \\
& \text { and for, } \quad \frac{3}{-(3-3 k)}=\frac{7}{5 k+1} \\
& \Rightarrow \quad 7(-3+3 k)=3(5 k+1) \\
& \Rightarrow \quad-21+21 k=15 k+3 \\
& \Rightarrow \quad 21 k-15 k=3+21 \\
& \Rightarrow \quad 6 k=24 \\
& \Rightarrow \quad k=4 \text {. }
\end{align*}
$$

and for,

$$
\frac{2}{k+2}=\frac{7}{5 k+1}
$$

$\Rightarrow \quad 7(k+2)=2(5 k+1)$
$\Rightarrow \quad 7 k+14=10 k+2$
$\Rightarrow \quad 7 k-10 k=2-14$
$\Rightarrow \quad-3 k=-12$
$\Rightarrow \quad k=4$
$1 / 2$
Hence, the value of $k$ is 4 , for which the given equations have infinitely many solutions.
[CBSE Marking Scheme, 2019]
Q. 5. For what value of $p$ will the following pair of linear equations have infinitely many solutions.

$$
(p-3) x+3 y=p
$$

and

$$
p x+p y=12
$$

$$
\mathrm{C}+\mathrm{U} \text { [CBSE SQP, 2018] }
$$

Sol. Given equation are:

$$
\begin{array}{rlrl} 
& & (p-3) x+3 y-p & =0  \tag{i}\\
\text { and } & p x+p y-12 & =0
\end{array}
$$

Comparing eq (i) with $a_{1} x+b_{1} y+c_{1}=0$
(ii) with $a_{2} x+b_{2} y+c_{2}=0$, we get.
$a_{1}=p-3, b_{1}=3, c_{1}=-p, a_{2}=p, b_{2}=p$ and $c_{2}=-12$
For infinitely many solutions,
and for,

$$
\frac{3}{p}=\frac{p}{12}
$$

$\Rightarrow \quad p^{2}=36$
$\Rightarrow \quad p= \pm 6$
and for, $\quad \frac{p-3}{p}=\frac{p}{12}$
$\Rightarrow \quad p^{2}=12 p-36$
$\Rightarrow \quad p^{2}-12 p+36=0$
$\Rightarrow \quad p^{2}-6 p-6 p+36=0$
$\Rightarrow \quad p(p-6)-6(p-6)=0$
$\Rightarrow \quad p=6,6$
Hence, the value of $p$ is 6 , for which the given equations have infinitely many solutions.
[CBSE Marking Scheme, 2018]

$$
\begin{aligned}
& \frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}} \\
& \text { i.e., } \quad \frac{p-3}{p}=\frac{3}{p}=\frac{-p}{-12} \\
& \text { For, } \quad \frac{p-3}{p}=\frac{3}{p} \\
& \Rightarrow \quad p^{2}-6 p=0 \\
& \Rightarrow \quad p(p-6)=0 \\
& p=0, p=6
\end{aligned}
$$

Q. 6. Find the value (s) of $k$ for which the pair of linear equations $k x+y=k^{2}$ and $x+k y=1$ have infinitely many solutions.

C + U [CBSE SQP, 2017]
Sol. Given equation are:

$$
\begin{array}{r}
k x+y-k^{2}=0  \tag{i}\\
x+k y-1=0
\end{array}
$$

and
Comparing eq. (i) with $a_{1} x+b_{1} y+c_{1}=0$ and eq. (ii) with $a_{2} x+b_{2} y+c_{2}=0$, we get
$a_{1}=k, b_{1}=1, c_{1}=-k^{2}, a_{2}=1, b_{2}=k$ and $c_{2}=-1$

For infinitely many solutions,
$1 / 2$

$$
\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}
$$

i.e.,

For
and for,

$$
\frac{1}{k}=\frac{k^{2}}{1}
$$

$\Rightarrow \quad k^{3}=1$
$\therefore \quad k=1$
$1 / 2$
and for,

$$
\frac{k}{1}=\frac{k^{2}}{1}
$$

$\Rightarrow \quad k^{2}=k$
$\Rightarrow \quad k^{2}-k=0$
$\Rightarrow \quad k(k-1)=0$
$\Rightarrow \quad k=0$ or 1
Hence, the value of $k$ is 1 , for which the given equations have infinitely many solutions.
Q.7. Given the linear equation $3 x+4 y=9$. Write another linear equation in these two variables such that the geometrical representation of the pair so formed is:
(i) intersecting lines
(ii) coincident lines.

R [Board Term-1, 2016]
Sol. (i) For intersecting lines $\frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}}$
So, one of the possible equation is $3 x-5 y=10 \quad \mathbf{1}$
(ii) For coincident lines $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}$

So, one of the possible equation is $6 x+8 y=18 \quad 1$
[CBSE Marking Scheme, 2016]
Q. 8. Find whether the lines represented by $2 x+y$ $=3$ and $4 x+2 y=6$ are parallel, coincident or intersecting.

U [Board Term-1, 2016]

Sol. Here, $a_{1}=2, b_{1}=1, c_{1}=-3$
and $a_{2}=4, b_{2}=2, c_{2}=-6$
If

$$
\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}
$$

Then the lines are coincident.
Clearly, $\quad \frac{2}{4}=\frac{1}{2}=\frac{3}{6}$ 1

Hence, lines are coincident.
[CBSE Marking Scheme, 2016]
Q. 9. Find whether the following pair of linear equations is consistent or inconsistent:

$$
\begin{array}{r}
3 x+2 y=8 \\
6 x-4 y=9
\end{array}
$$

and
U [Board Term-1, 2016]
Sol. Since,

$$
\frac{3}{6} \neq \frac{2}{-4}
$$

i.e., $\quad \frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}}$

Hence, the pair of linear equations is consistent. 1
[CBSE Marking Scheme, 2016]

## Short Answer Type Questions-II

## 3 marks each

Q. 1. Determine graphically the coordinates of the vertices of triangle, the equations of whose sides are given by $2 y-x=8,5 y-x=14$ and $y-2 x=1$.

A + E [CBSE Delhi Set-I, 2020]
Sol. Given, $\quad 2 y-x=8$

| $\Rightarrow \quad x=2 y-8$ |  |  |  |
| :---: | :---: | :---: | :---: |
| $y$ | 0 | 4 | 5 |
| $x=2 y-8$ | -8 | 0 | 2 |
| $\begin{aligned} & & 5 y-x & =14 \\ \Rightarrow & & x & =5 y-14 \end{aligned}$ |  |  |  |
| $y$ | 3 | 4 | 2 |
| $x=5 y-14$ | 1 | 6 | -4 |
| $\begin{array}{rlrl} \text { and } & y-2 x & =1 \\ \Rightarrow & & y & =1+2 x \end{array}$ |  |  |  |
| $x$ | 0 | 1 | 2 |
| $y=1+2 x$ | 1 | 3 | 5 |

$\Rightarrow \quad x=5 y-14$

Plotting the above points and drawing lines joining them, we get the graphical representation:


Hence, the coordinates of the vertices of the triangle $A B C$ are $A(1,3), B(2,5)$ and $C(-4,2)$.
[CBSE Marking Scheme, 2020]
Q. 2. Three lines $x+3 y=6,2 x-3 y=12$ and $x=0$ are enclosing a beautiful triangular park. Find the points of intersection of the lines graphically and the area of the park, if all measurements are in km .

AE [Board Term-1, 2015]

Sol.
and $\quad \begin{aligned} x+3 y & =6 \\ 2 x-3 y & =12\end{aligned}$
For equation (i): $\quad y=\frac{6-x}{3}$

| $x$ | 6 | 0 | 3 |
| :--- | :--- | :--- | :--- |
| $y$ | 0 | 2 | 1 |

$\mathrm{A}(6,0), \mathrm{B}(0,2)$
Equation (ii): $\quad y=\frac{2 x-12}{3}$

| $x$ | 6 | 0 | 3 |
| :---: | :---: | :---: | :---: |
| $y$ | 0 | -4 | -2 |

$\mathrm{A}(6,0), \mathrm{C}(0,-4)$
1
Vertices $A(6,0), B(0,2)$ and $C(0,-4)$
$\therefore \quad$ Area of $\triangle A B C=\left(\frac{1}{2} \times 2 \times 6+\frac{1}{2} \times 4 \times 6\right) \mathrm{km}^{2}$
$=(6+12) \mathrm{km}^{2}$
$=18 \mathrm{~km}^{2}$

Q. 3. Represent the following pair of linear equations graphically and hence comment on the condition of consistency of this pair.

$$
x-5 y=6,2 x-10 y=12
$$

Sol. Given,

$$
x-5 y=6
$$

$$
\Rightarrow \quad y=\frac{x-6}{5}
$$

| $x$ | 6 | 1 | -4 |
| :---: | :---: | :---: | :---: |
| $y$ | 0 | -1 | -2 |

and $\quad 2 x-10 y=12$
$\Rightarrow \quad y=\frac{x-6}{5}$

| $x$ | 6 | 1 | -4 |
| :---: | :---: | :---: | :---: |
| $y$ | 0 | -1 | -2 |



Since, the lines are coincident, so the system of linear equations is consistent with infinitely many solutions.

## Long Answer Type Questions

## 5 marks each

[AI Q. 1. For what values of $m$ and $n$ the following system of linear equations has infinitely many solutions.
$3 x+4 y=12$
and $(m+n) x+2(m-n) y=5 m-1$
A [CBSE Comp. Set I/II/IIII 2018]
[Board Term-1, 2015]
Sol. For infinitely many solutions,

$$
\begin{array}{rlrl}
\frac{3}{m+n} & =\frac{4}{2(m-n)}=\frac{-12}{-(5 m-1)} \\
& \frac{3}{m+n} & =\frac{4}{2(m-n)} \\
\Rightarrow & \frac{4}{2(m-n)} & =\frac{12}{5 m-1} \\
\Rightarrow & m-6 n & =-1 \tag{2}
\end{array}
$$

Solving (1) and (2), we get, $m=5$ and $n=1 \quad 1$
[CBSE Marking Scheme, 2018]

## COMMONLY MADE ERROR

() Many candidates are not able to solve this problem because of not having the basic idea of unique solution, infinitely many solutions and no solution.

## ANSWERING TIP

- Candidates must be familiar with all three condition for solvability like unique solution, infinitely many solutions and no solution.
Q. 2. For Uttarakhand flood victims two sections $A$ and $B$ of class $X$ contributed $₹ \mathbf{1 , 5 0 0}$. If the contribution of X-A was ₹ 100 less than that of X-B, find graphically the amounts contributed by both the sections.

A [Board Term-1, 2016]

Sol. Let amounts contributed by two sections X-A and X-B be ₹ $x$ and $₹ y$.

$$
\begin{align*}
& x+y & =1,500  \tag{i}\\
\text { and } & y-x & =100 \tag{ii}
\end{align*}
$$

From (i),

$$
y=1500-x
$$

From (ii),

$$
\begin{equation*}
y=100+x \tag{1}
\end{equation*}
$$

Point of intersection $=(700,800)$
Hence, X-A contributed ₹ 700 and X-B contributed = ₹ 800 .
[CBSE Marking Scheme, 2016] 1
Q. 3. Determine graphically whether the following pair of linear equations:

$$
3 x-y=7
$$

and $\quad 2 x+5 y+1=0$, has:
(i) a unique solution or
(ii) infinitely many solutions or
(iii) no solution.

A [Board Term-1, 2015]
Sol. Given equations are:

$$
\begin{align*}
3 x-y & =7  \tag{i}\\
\text { and } \quad 2 x+5 y+1 & =0 \tag{ii}
\end{align*}
$$



Since, point of intersection is $(2,-1)$. Hence, it has unique solution.
Hence, $x=2$ and $y=-1$.
Q.4. Draw the graphs of the pair of linear equations:

$$
x+2 y=5
$$

and

$$
2 x-3 y=-4
$$

Also, find the points where the lines meet the X-axis.

A [Board Term-1, 2014, 2015]
Sol. Given, $\quad x+2 y=5$
$\Rightarrow \quad y=\frac{5-x}{2}$


| $x$ | 1 | 3 | 5 |
| :--- | :--- | :--- | :--- |
| $y$ | 2 | 1 | 0 |

and

$$
2 x-3 y=-4
$$

$$
\Rightarrow \quad y=\frac{2 x+4}{3}
$$

| $x$ | -2 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| $y$ | 0 | 2 | 4 |

1
Thus, the lines meet X -axis at $(5,0)$ and $(-2,0)$ respectively. [CBSE Marking Scheme, 2015] 2
Q. 5. Solve graphically the pair of linear equations:

$$
\begin{array}{r}
3 x-4 y+3=0 \\
3 x+4 y-21=0 .
\end{array}
$$

and
Find the co-ordinates of the vertices of the triangular region formed by these lines and X-axis. Also, calculate the area of this triangle.

A [Board Term-1, 2015]
Sol. Given, $\quad 3 x-4 y+3=0$

$$
\Rightarrow \quad y=\frac{3 x+3}{4}
$$

| $x$ | 3 | 7 | -1 |
| :---: | :---: | :---: | :---: |
| $y$ | 3 | 6 | 0 |

and

$$
3 x+4 y-21=0
$$

$$
\Rightarrow \quad y=\frac{21-3 x}{4}
$$

| $x$ | 3 | 7 | 11 |
| :---: | :---: | :---: | :---: |
| $y$ | 3 | 0 | -3 |


(i) These lines intersect each other at point $(3,3)$. Hence, $x=3$ and $y=3$.
(ii) The vertices of triangular region are $(3,3),(-1,0)$ and $(7,0)$.
(iii)

$$
\text { Area of } \Delta=\frac{1}{2} \times 8 \times 3
$$

Hence, Area of obtained $\Delta=12$ sq. units.

## TOPIC - 2 <br> Algebraic Methods to Solve Pair of Linear Equations

## Revision Notes

> Algebraic Method: We can solve the linear equations algebraically by substitution method, elimination method and cross-multiplication method.

1. Substitution Method:
(i) Find the value of one variable (say $y$ ) in terms of the other variable i.e., $x$ from either of the equations.
(ii) Substitute this value of $y$ in other equation and reduce it to an equation in one variable.
(iii) Solve the equation so obtained and find the value of $x$.
(iv) Put this value of $x$ in one of the equations to get the value of variable $y$.
2. Elimination Method:
(i) Multiply given equations with suitable constants, make either the $x$-coefficients or the $y$-coefficients of the two equations equal.
(ii) Subtract or add one equation from the other to get an equation in one variable.
(iii) Solve the equation so obtained to get the value of the variable.
(iv) Put this value in any one of the equation to get the value of the second variable.

## Note:

(a) If in step (ii), we obtain a true equation involving no variable, then the original pair of equations has infinitely many solutions.
(b) If in step (ii), we obtain a false equation involving no variable, then the original pair of equations has no solution i.e., it is inconsistent.
3. Cross-multiplication Method: If two simultaneous linear equations $a_{1} x+b_{1} y+c_{1}=0$ and $a_{2} x+b_{2} y+c_{2}=0$ are given, then a unique solution is given by:

$$
\frac{x}{b_{1} c_{2}-b_{2} c_{1}}=\frac{y}{c_{1} a_{2}-c_{2} a_{1}}=\frac{1}{a_{1} b_{2}-a_{2} b_{1}}
$$

Then, $x=\frac{b_{1} c_{2}-b_{2} c_{1}}{a_{1} b_{2}-a_{2} b_{1}}$ and $y=\frac{c_{1} a_{2}-c_{2} a_{1}}{a_{1} b_{2}-a_{2} b_{1}}$
Note: To obtain the above result, following diagram may be helpful:


The arrows between the two numbers indicate that they are to be multiplied. The product with upward arrows are to be subtracted from the product with downward arrows.
$>$ Equations reducible to a pair of Linear Equations in two variables: Sometimes, a pair of equations in two variables is not linear but can be reduced to linear form by making some suitable substitutions. Here, first we find the solution of new pair of linear equations and then find the solution for the given pair of equations.

Steps to be followed for solving word problems

| S. No. | Problem type | Steps to be followed |
| :---: | :---: | :--- |
| $\mathbf{1 .}$ | Age Problems | If the problem involves finding out the ages of two persons, take the present <br> age of one person as $x$ and of the other as $y$. Then, ' $a$ ' years ago, age of $1^{\text {st }}$ <br> person was ' $x-a^{\prime}$ years and that of $2^{\text {nd }}$ person was ' $y-a^{\prime}$ and after ' $b^{\prime}$ years, <br> age of $1^{\text {st }}$ person will be' $x+b^{\prime}$ years and that of $2^{\text {nd }}$ person will be ' $y+b^{\prime}$ years. <br> Formulate the equations and then solve them. |

$\left.\begin{array}{|c|l|l|}\hline \text { 2. } & \begin{array}{l}\text { Problems based on } \\ \text { Numbers and Digits }\end{array} & \begin{array}{l}\text { Let the digit in unit's place be } x \text { and that in ten's place be } y . \text { The two-digit } \\ \text { number is given by } 10 y+x . \text { On interchanging the positions of the digits, the } \\ \text { digit in unit's place becomes } y \text { and in ten's place becomes } x . \text { The two digit } \\ \text { number becomes } 10 x+y . \\ \text { Formulate the equations and then solve them. }\end{array} \\ \hline \text { 3. } & \begin{array}{l}\text { Problems based on } \\ \text { Fractions }\end{array} & \begin{array}{l}\text { Let the numerator of the fraction be } x \text { and denominator be } y, \text { then the fraction } \\ \text { is } \frac{x}{y} . \\ \text { Formulate the linear equations on the basis of conditions given and solve for } x \\ \text { and } y \text { to get the value of the fraction. }\end{array} \\ \hline \text { 4. } & \begin{array}{l}\text { Problems based on } \\ \text { Distance, Speed and } \\ \text { Time }\end{array} & \begin{array}{l}\text { Speed }=\frac{\text { Distance }}{\text { Time }}\end{array} \\ \text { or Distance }=\text { Speed } \times \text { Time and Time }=\frac{\text { Distance }}{\text { Speed }} . \\ \text { To solve the problems related to speed of boat going downstream and upstream, } \\ \text { let the speed of boat in still water be } x \text { km/h and speed of stream be } y \text { km } / h . \text { Then, } \\ \text { the speed of boat in downstream }=(x+y) \text { km/h and speed of boat in upstream } \\ \text { ( } x-y) \text { km/h. }\end{array}\right\}$

## How is it done on the GREENBOARD?

Q.l. Solve the following system of equations by substitution method.

$$
x+2 y=5 \text { and } 2 x+y=4
$$

## Solution:

## Step I: Given equations are

$$
\begin{align*}
& \quad \begin{array}{l}
x+2 y=5 \\
\text { and } 2 x+y=4 \\
\text { Step II: From } x+2 y=5 \text {, we get } \\
x=5-2 y \quad . . \\
\text { Step III: Putting this value of } x \text { in } \\
\text { other equation }
\end{array} . \tag{i}
\end{align*}
$$

$$
\begin{aligned}
2 x+y & =4, \text { we get } \\
2(5-2 y)+y & =4 \\
10-4 y+y & =4 \\
3 y & =10-4 \\
y & =\frac{6}{3}=2
\end{aligned}
$$

Setp IV: On putting this value of $y$ in eq. (iii)
Also $\quad x=5-2(2)$
$x=1$
$x=1$ and $y=2$.

## Very Short Answer Type Questions

## 1 mark each

(AI) Q. 1. If 3 chairs and 1 table costs ₹ 1500 and 6 chairs and 1 table costs ₹ 2400 . Form linear equations to represent this situation.

U + C [CBSE SQP, 2020-21]
Sol. Let the cost of 1 chair be ₹ $x$ and the cost of 1 table be ₹ $y$, then cost of 3 chairs + cost of 1 table $=1500$ $\Rightarrow \quad 3 x+y=1500 \quad 1 / 2$ Similarly, Cost of 6 chairs + cost of 1 table $=2400$ $\Rightarrow \quad 6 x+y=2400$. $1 / 2$
[CBSE Marking Scheme, 2021]
Q. 2. If $x=a$ and $y=b$ is the solution of the pair of equations $x-y=2$ and $x+y=4$, find the values of $a$ and $b$.

U [CBSE Comptt. Set I/II/III, 2018]
Sol. Solving for $x$ and $y$ and getting $x=3$ and $y=1$ $\therefore a=3$ and $b=1$.
[CBSE Marking Scheme, 2018]

## Detailed Solution:

Given equations are:

$$
\begin{align*}
& x-y & =2  \tag{i}\\
\text { and } & x+y & =4 \tag{ii}
\end{align*}
$$

Adding eq. (i) and (ii), we get

$$
2 x=6
$$

$\Rightarrow \quad x=3 \quad 1 / 2$
Substituting $x=3$ in eq. (ii), we get

$$
3+y=4
$$

$\Rightarrow \quad y=4-3=1$
If $x=a$ and $y=b$ is the solution of given equations, then.

$$
a=x=3 \text { and } b=y=1 . \quad 1 / 2
$$

Hence, $\quad a=3$ and $y=1$.
Q. 3. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is $₹ 75$. then find the number of $₹ 1$ and $₹ 2$ coins are, respectively
Sol. Let the number of $₹ 1$ coins $=x$ and the number of $₹ 2$ coins $=y$
So, according to the question,

$$
\begin{array}{r}
x+y=50 \\
x+2 y=75 \tag{ii}
\end{array}
$$

Subtracting equation (i) from (ii),

$$
y=25
$$

Substituting value of $y$ in (i),

$$
x=25
$$

So, $y=25$ and $x=25$.

## Short Answer Type Questions-I

## 2 marks each

Q. 1. The larger of two supplementary angles exceed the smaller by $18^{\circ}$. Find the angles.

A [CBSE Outside Delhi-I, 2019]
Sol. Let larger angle be $x^{\circ}$
$\therefore$ Smaller angle $=180^{\circ}-x^{\circ}$
$\therefore(x)-(180-x)=18$
$\therefore$ The two angles are $99^{\circ}, 81^{\circ}$
[CBSE Marking Scheme, 2019]

## Detailed Solution:

Let one angle be $x$.
Then, other angle (it's supplementary angle) $=\left(180^{\circ}-x\right)$
Given,

$$
x+18^{\circ}=\left(180^{\circ}-x\right)
$$

$$
\Rightarrow \quad 2 x=180^{\circ}-18^{\circ}
$$

$$
\Rightarrow \quad 2 x=162^{\circ}
$$

$$
\begin{equation*}
\Rightarrow \quad x=81^{\circ} \tag{1}
\end{equation*}
$$

Now, $\quad$ other angle $=180^{\circ}-81^{\circ}$

$$
\begin{equation*}
=99^{\circ} \tag{1}
\end{equation*}
$$

Hence, two required angles are $81^{\circ}$ and $99^{\circ}$.
Q. 2. Sumit is 3 times as old as his son. Five years later, he shall be two and a half times as old as his son. How old is Sumit at present ?

A [CBSE Outside Delhi-I, 2019]

Sol. Let Son's present age be $x$ years
Then Sumit's present age $=3 x$ years.
$1 / 2$
$\therefore 5$ Years later, we have, $3 x+5=\frac{5}{2}(x+5) \quad 1 / 2$
$6 x+10=5 x+25 \Rightarrow x=15 \quad 1 / 2$
$\therefore$ Sumit's present age $=45$ years
$1 / 2$
[CBSE Marking Scheme, 2019]

## Detailed Solution:

Let Sumit's present age be $x$ years and his son's present age be $y$ years.
According to given condition,

$$
\begin{equation*}
x=3 y \tag{i}
\end{equation*}
$$

After five years,

$$
\text { Sumit's age }=x+5
$$

and $\quad$ His son's age $=y+5$
Now, again according to given condition,

$$
\begin{array}{rlrl}
x+5 & =2 \frac{1}{2}(y+5) \\
\Rightarrow & x+5 & =\frac{5}{2}(y+5) \\
\Rightarrow & 2(x+5) & =5(y+5) \\
\Rightarrow & 2 x+10 & =5 y+25
\end{array}
$$

$$
\begin{aligned}
\Rightarrow & 2 x & =5 y+15 \\
\Rightarrow & 2(3 y) & =5 y+15 \quad \text { [from eq (i)] } \\
\Rightarrow & 6 y & =5 y+15 \\
\Rightarrow & y & =15
\end{aligned}
$$

Q. 3. Solve below simultaneous equations for $x$ and $y$,

$$
\begin{array}{ll} 
& 3 x-5 y=4 \\
\text { and } & 9 x-2 y=7 .
\end{array}
$$

$$
\begin{array}{ll}
\text { Again, from eq. (i), } \\
& x=3 y \\
\Rightarrow & x=3 \times 15=45
\end{array}
$$

Hence, Sumit's present age is 45 years.

Topper Answer, 2019

Sol. I Given -


AI Q. 4. In Fig., ABCD is a rectangle. Find the values of $x$ and $y$.


A [CBSE Delhi \& OD, 2018]

Sol. Since,

$$
\begin{aligned}
A B & =C D \text { and } B C=A D \\
x+y & =30 \\
x-y & =14
\end{aligned}
$$

$$
\text { Solving to get } x=22 \text { and } y=8
$$


[AI] Q. 5. The incomes of two persons $A$ and $B$ are in the ratio 8: 7 and the ratio of their expenditures is 19: 16 If their savings are ₹ 2550 per month, find their monthly income. A; E [Board Term-1, 2016]
Sol. Let income of $A=8 x$ and income of $B=7 x$.
Also their expenditures be $19 y$ and $16 y$.

$$
\begin{array}{ll}
\Rightarrow & 8 x-19 y=2550  \tag{i}\\
\text { and } & 7 x-16 y=2550
\end{array}
$$

Solving the equations

$$
\begin{align*}
x & =1530 \text { and } y=510 \\
A & =12240  \tag{1}\\
B & =10710
\end{align*}
$$

$\therefore$ Salary of $\quad A=12240$
Salary of
[AI) Q. 6. Solve the following pair of linear equations by cross multiplication method:

$$
\begin{array}{r}
x+2 y=2 \\
x-3 y=7
\end{array}
$$

U [Board Term-1, 2015, 2016]
Sol.

$$
\begin{array}{r}
x+2 y-2=0 \\
x-3 y-7=0
\end{array}
$$

Using the formula

$$
\begin{array}{cc} 
& \frac{x}{b_{1} c_{2}-b_{2} c_{1}}=\frac{y}{c_{1} a_{2}-c_{2} a_{1}}=\frac{1}{a_{1} b_{2}-a_{2} b_{1}} \\
& \frac{x}{-14-6}=\frac{y}{-2+7}=\frac{1}{-3-2} \\
\Rightarrow & \frac{x}{-20}=\frac{y}{5}=\frac{-1}{5} \\
\Rightarrow & \\
& \frac{x}{-20}=\frac{-1}{5}
\end{array}
$$

$\Rightarrow \quad \frac{y}{5}=\frac{-1}{5}$
$\Rightarrow \quad x=4$
and $\quad y=-1$
[CBSE Marking Scheme, 2016]
Q. 7. Solve the following pair of linear equations by substitution method:
and $\quad \begin{aligned} 3 x+2 y-7 & =0 \\ 4 x+y-6 & =0\end{aligned}$
U [Board Term-1, 2015]
Sol. Given, $\quad 3 x+2 y-7=0$
and $\quad 4 x+y-6=0$
From eqn. (ii), we have

$$
\begin{equation*}
y=6-4 x \tag{ii}
\end{equation*}
$$

On putting this value of $y$ in eqn. (i), we get

$$
\begin{align*}
3 x+2(6-4 x)-7 & =0  \tag{iii}\\
3 x+12-8 x-7 & =0 \\
5-5 x & =0 \\
\therefore \quad 5 x & =5 \\
\therefore \quad x & =1
\end{align*}
$$

Substituting this value of $x$ in (iii), we get

$$
\begin{align*}
& y=6-4 \times 1 \\
& y=2
\end{align*}
$$

Hence, values of $x$ and $y$ are 1 and 2 respectively. $1 / 2$
Q. 8. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Shristi paid ₹ 27 for a book kept for seven days, while Rekha paid ₹ 21 for the book she kept for five days. Find the fixed charge and the additional charge paid by them.

C [Board Term-1, 2015]

Sol. Let fixed charges for reading book $=₹ x$
Let additional charges per day $=₹ y$ then

$$
\begin{align*}
& x+4 y=27  \tag{i}\\
& x+2 y=21 \tag{ii}
\end{align*}
$$

On solving both the equations

$$
x=₹ 15 \text { and } y=₹ 3
$$

Hence, Shristi paid additional charges $=₹ 12$
Rekha paid additional charges $=₹ 6$

## Short Answer Type Questions-II

## 3 marks each

Q. 1. In the figure, $A B C D E$ is a pentagon with $B E \| C D$ and $B C|\mid D E . B C$ is perpendicular to $C D$.
$A E=A B=5 \mathrm{~cm}, B E=7 \mathrm{~cm}, B C=x-y$ and $C D=$ $x+y$. If the perimeter of $A B C D E$ is 27 cm . find the value of $x$ and $y$, given $x, y \neq 0$.


A [CBSE SQP, 2020]
Sol. $x+y=7$ and $2(x-y)+x+y+5+5=27$
$\therefore x+y=7$ and $3 x-y=17$
Solving, we get, $x=6$ and $y=1$
[CBSE SQP Marking Scheme, 2020]

## Detailed Solution:

Here, BCDE is a rectangle.
$\therefore \quad x+y=7$
Perimeter of $A B C D E=27 \mathrm{~cm}$ (given)


$$
\begin{align*}
& \therefore A B+B C+C D+D E+E A=27 \\
& \Rightarrow \\
& \Rightarrow \quad 3 x-y=27-10  \tag{ii}\\
& 3 x-y=17
\end{align*}
$$

Adding eq. (i) and (ii), we get

$$
\begin{array}{rlrl} 
& & 4 x & =24 \\
\Rightarrow & x & =6 .
\end{array}
$$

Substituting the value of $x$ in eq. (i), we get

$$
\begin{aligned}
6+y & =7 \\
y & =7-6=1
\end{aligned}
$$

Hence, $x=6$ and $y=1$.
[8I) Q. 2. Solve the following system of equations:

$$
\begin{aligned}
& \frac{21}{x}+\frac{47}{y}=110 \\
& \frac{47}{x}+\frac{21}{y}=162, x, y \neq 0 .
\end{aligned}
$$

A [CBSE SQP, 2020]

Sol. Let $\frac{1}{x}=a$ and $\frac{1}{y}=b$
$\Rightarrow 21 a+47 b=110$ and $47 a+21 b=162$
Adding and subtracting the two equations, we get
$a+b=4$ and $a-b=2$
Solving the above two equations, we get $a=3$ and b = 1

$$
x=\frac{1}{3} \text { and } y=1
$$

[CBSE SQP Marking Scheme, 2020] 3
Detailed Solution:
Given equations are:

$$
\begin{aligned}
& \frac{21}{x}+\frac{47}{y} & =110 \\
\text { and } & \frac{47}{x}+\frac{21}{y} & =162
\end{aligned}
$$

Let $\frac{1}{x}=a$ and $\frac{1}{y}=b$, then

$$
\begin{align*}
& 21 a+47 b=110  \tag{i}\\
& 47 a+21 b=162 \tag{ii}
\end{align*}
$$

and $\quad 47 a+21 b=162$
Adding eq. (i) and (ii), we get

$$
\begin{align*}
& & 68 a+68 b & =272 \\
\Rightarrow & & 68(a+b) & =272 \\
\Rightarrow & & a+b & =4 \tag{iii}
\end{align*}
$$

Subtracting eq. (i) from (ii), we get

$$
\begin{align*}
& & 26 a-26 b & =52 \\
\Rightarrow & & 26(a-b) & =52 \\
\Rightarrow & & a-b & =2 \tag{iv}
\end{align*}
$$

Adding eq. (iii) and (iv), we get

$$
2 a=6
$$

$\Rightarrow \quad a=3$
Substituting $a=3$ in eq. (iii), we get

$$
\begin{aligned}
& & 3+b & =4 \\
\Rightarrow & & b & =4-3=1
\end{aligned}
$$

$$
\text { If, } a=3 \text {, then } \frac{1}{x}=3 \Rightarrow x=\frac{1}{3}
$$

If, $b=1$, then $\frac{1}{y}=1 \Rightarrow y=1$.
(AI) Q. 3. A man can row a boat downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water. Also find the speed of the stream.

A [CBSE Delhi, Set-III, 2020]
Sol. Let the speed of the boat in still water be $x \mathrm{~km} / \mathrm{hr}$ and speed of the stream be $y \mathrm{~km} / \mathrm{hr}$.
$\therefore$ Relative Speed of boat in upstream

$$
=(x-y) \mathrm{km} / \mathrm{hr}
$$

and Relative speed of boat in downstream

$$
\begin{equation*}
=(x+y) \mathrm{km} / \mathrm{hr} \tag{1}
\end{equation*}
$$

According to question,

$$
\begin{align*}
& & \frac{20}{x+y} & =2 \\
\Rightarrow & & x+y & =10  \tag{i}\\
\text { and } & & \frac{4}{x-y} & =2 \\
\Rightarrow & & x-y & =2
\end{align*}
$$

On adding eq. (i) and (ii), we get

$$
\begin{aligned}
& & 2 x & =12 \\
\Rightarrow & & x & =6
\end{aligned}
$$

Putting the value of $x$ is eq. (i),

$$
6+y=10
$$

$$
\Rightarrow \quad y=10-6=4
$$

Hence, Speed of a boat in still water $=6 \mathrm{~km} / \mathrm{hr}$ and $\quad$ speed of the stream $=4 \mathrm{~km} / \mathrm{hr} . \quad \mathbf{1}$
Q. 4. If $2 x+y=23$ and $4 x-y=19$, find the value of $(5 y-2 x)$ and $\left(\frac{y}{x}-2\right)$.

U [CBSE, OD Set-I, 2020]
Sol. Given,

$$
\begin{equation*}
2 x+y=23 \tag{i}
\end{equation*}
$$

and $\quad 4 x-y=19$
On adding eq. (i) and (ii), we get

$$
\begin{equation*}
6 x=42 \Rightarrow x=7 \tag{ii}
\end{equation*}
$$

Putting the value of $x$ in eq. (i), we get

$$
\begin{array}{rlrl}
\Rightarrow & & 14+y & =23 \\
y & =23-14=9 \\
\text { Hence, } & & 5 y-2 x & =5 \times 9-2 \times 7=45-14 \\
& & =31 .
\end{array}
$$

and $\frac{y}{x}-2=\frac{9}{7}-2=\frac{9-14}{7}=\frac{-5}{7}$.
Q. 5. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father. A [CBSE Delhi-Set- I, II, III, 2019]

Sol. Let sum of the ages of two children be $x$ years and father's age be $y$ years.

$$
\begin{array}{lrl}
\therefore & y & =3 x \\
\text { and } & y+5 & =2(x+10) \tag{1}
\end{array}
$$

Solving equations (1) and (2)

$$
\text { and } \quad y=45
$$

Father's present age is 45 years.
[CBSE Marking Scheme, 2019]

## Detailed Solution:

Let the age of father be $x$ years and sum of the ages of his children be $y$ years.
After 5 years,
Father's age $=(x+5)$ years
Sum of ages of his children $=(y+10)$ years $\quad 1 / 2$
According to the given condition,

$$
\begin{equation*}
x=3 y \tag{i}
\end{equation*}
$$

and $\quad x+5=2(y+10)$
or, $\quad x-2 y=15$
Solving eq's (i) and (ii), we get

$$
\begin{align*}
& 3 y-2 y
\end{align*}=15
$$

Substituting value of $y$ in eq. (i), we get

$$
x=3 \times 15=45
$$

Hence, father's present age is 45 .

## COMMONLY MADE ERROR

- Mostly students get confused in these type of problems as they face difficulty in forming the equations from statement. Sometimes they form equations incorrectly.


## ANSWERING TIP

- Practice more such questions and get the concept cleared.
Q.6. A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction. A [CBSE Delhi Set-I, II, III, 2019]

Sol. Let the fraction be $\frac{x}{y}$

$$
\begin{equation*}
\therefore \quad \frac{x-2}{y}=\frac{1}{3} \tag{1}
\end{equation*}
$$

and $\quad \frac{x}{y-1}=\frac{1}{2}$
Solving (1) and (2) to get $x=7, y=15$.
$\therefore$ Required fraction is $\frac{7}{15}$
[CBSE Marking Scheme, 2019]

## Detailed Solution:

Let the fraction be $\frac{x}{y}$.
According to the first condition,

$$
\begin{align*}
& & \frac{x-2}{y} & =\frac{1}{3} \\
\Rightarrow & & 3 x-6 & =y \\
\Rightarrow & & y & =3 x-6 \tag{i}
\end{align*}
$$

According to the second condition,

$$
\begin{array}{rlrl} 
& & \frac{x}{y-1} & =\frac{1}{2} \\
\Rightarrow & & 2 x & =y-1 \\
\Rightarrow & y & =2 x+1 \tag{ii}
\end{array}
$$

From eq's. (i) and (ii), we get

$$
\begin{align*}
& & 3 x-6 & =2 x+1 \\
\Rightarrow & & x & =7
\end{align*}
$$

Substituting value of $x$ in eq (i), we get

$$
\begin{align*}
& y=3(7)-6 \\
& y=21-6=15 \tag{1}
\end{align*}
$$

Hence, fraction is $\frac{7}{15}$.
Q. 7. Places A and B are 80 km apart from each other on a highway. A car starts from A and another from B at the same time. If they move in same direction they meet in 8 hours and if they move towards each other they meet in 1 hour 20 minutes. Find the speed of cars.

A [CBSE SQP 2018]
Sol. Let the speed of car at A be $x \mathrm{~km} / \mathrm{h}$
And the speed of car at B $y \mathrm{~km} / \mathrm{h}$
Case $1 \quad 8 x-8 y=80$
or, $\quad x-y=10$
Case $2 \frac{4}{3} x+\frac{4}{3} y=80$
or, $\quad x+y=60$
On solving, $x=35$ and $y=25$
Hence, speed of cars at A and B are $35 \mathrm{~km} / \mathrm{h}$ and $25 \mathrm{~km} / \mathrm{h}$ respectively.
[CBSE Marking Scheme, 2018]

## Detailed Solution:

Let the speed of the car 1 from A be $x \mathrm{~km} / \mathrm{h}$ and speed of the car 2 from B be $y \mathrm{~km} / \mathrm{h}$

## Same direction:

Distance covered by car $1=80+$ (distance covered by car 2 )

$$
\begin{align*}
\Rightarrow & 8 x & =80+8 y \\
\Rightarrow & 8 x-8 y & =80 \\
\Rightarrow & x-y & =10 \tag{i}
\end{align*}
$$

Opposite direction:
Distance covered by car $1+$ distance covered by car 2

$$
\begin{align*}
& =80 \mathrm{~km} \\
\Rightarrow \quad \frac{4}{3} x+\frac{4}{3} y & =80 \\
x+y & =60 \tag{ii}
\end{align*}
$$

Adding eq. (i) and (ii), we get

$$
\begin{array}{r}
2 x=70 \\
x=35
\end{array}
$$

substituting $x=35$ in eq. (i)

$$
y=25
$$

$\therefore$ Speed of the car 1 from A $=35 \mathrm{~km} / \mathrm{h}$
and speed of the car 2 of from $B=25 \mathrm{~km} / \mathrm{h}$

## COMMONLY MADE ERROR

Some candidates, are not able to frame this word problem into equation.

## ANSWERING TIP

Emphasis on solving such type of application based problem.
[AI Q. 8. A part of monthly hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay ₹ 3,000 as hostel charges whereas Mansi who takes food for 25 days has to pay ₹ 3,500 as hostel charges. Find the fixed charges and the cost of food per day.

## C [Board Term-1, 2015-16]

Sol. Let fixed charge be $x$ and per day food cost be $y$

$$
\begin{equation*}
\text { Then, } \quad x+20 y=3000 \tag{i}
\end{equation*}
$$

and $\quad x+25 y=3500$
Subtracting (i) from (ii), we get

$$
\begin{array}{r}
x+25 y=3500  \tag{ii}\\
x+20 y=3000 \\
-\quad- \\
\hline 5 y=500 \\
\hline y=100
\end{array}
$$

Substituting this value of $y$ in (i), we get

$$
\begin{array}{rlrl} 
& x+20(100) & =3000 \\
x & =1000 \\
\therefore \quad & x & =1000 \text { and } y=100
\end{array}
$$

Hence, fixed charge and cost of food per day are $₹ 1,000$ and $₹ 100$.
[ AI I) Q. 9. Solve for $x$ and $y$ :

$$
\frac{x}{2}+\frac{2 y}{3}=-1
$$

and

$$
x-\frac{y}{3}=3
$$

U [Board Term-1, 2015]
Sol. Given, $\quad \frac{x}{2}+\frac{2 y}{3}=-1$

$$
\begin{array}{rlrl} 
& \frac{3 x+4 y}{6} & =-1 \\
\text { or, } & 3 x+4 y & =-6 \\
\text { and } & \frac{x}{1}-\frac{y}{3} & =3 \\
& \frac{3 x-y}{3} & =3 \\
& \text { or, } & 3 x-y & =9 \tag{ii}
\end{array}
$$

On subtracting eqn. (ii) from eqn. (i),

$$
\begin{aligned}
3 x+4 y & =-6 \\
3 x-y & =9 \\
-++ & - \\
\hline 5 y & =-15 \\
\therefore \quad y & =-3
\end{aligned}
$$

Putting $y=-3$ in eq (i), we get

$$
\begin{aligned}
3 x+4(-3) & =-6 \\
3 x-12 & =-6 \\
3 x & =12-6 \\
& \\
\therefore \quad \quad \quad \quad \quad x & =6 \\
x & =2
\end{aligned}
$$

Hence, $x=2$ and $y=-3$.
Q. 10. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their respective ages.

A [Board Term-1, 2015]
Sol. Let age of father and son be $x$ and $y$ respectively.

$$
\begin{array}{lr}
\text { Then, } & x+y=40 \\
\text { and } & x=3 y \tag{ii}
\end{array}
$$

By solving eqns. (i) and (ii), we get

$$
\begin{equation*}
x=30 \text { and } y=10 \tag{1}
\end{equation*}
$$

Thus, the ages of father and son are 30 years and 10 years.
[CBSE Marking Scheme, 2015]
Q. 11. Raghav scored 70 marks in a test, getting 4 marks for each right answer and losing 1 mark for each wrong answer. Had 5 marks been awarded for each correct answer and 2 marks been deducted for each wrong answer, then Raghav would have scored 80 marks. How many questions were there in the test?

C [Board Term-1, 2015]
Sol. Let number of right answers be $x$.
Let number of wrong answers be $y$.
As per question

$$
\begin{array}{r}
4 x-y=70 \\
5 x-2 y=80 \tag{ii}
\end{array}
$$

$2 \times$ eq. (i) - eq. (ii),

$$
\begin{gather*}
8 x-2 y=140 \\
5 x-2 y=80 \\
-\quad+\quad-  \tag{2}\\
\hline 3 x=60
\end{gather*}
$$

$\Rightarrow \quad x=20$
Substituting the value of $x$ in eq (i) to get value of $y$,

$$
\begin{array}{rlrl} 
& & 4(20)-y & =70 \\
\Rightarrow & 80-y & =70 \\
\therefore & y & =10 \tag{1}
\end{array}
$$

Hence, total number of questions are $=20+10=30$.
Q. 12. In a painting competition of a school a child made Indian national flag whose perimeter was 50 cm . Its area will be decreased by 6 square cm , if length is decreased by 3 cm and breadth is increased by 2 cm then find the dimension of flag.
[Board Term-1, 2015]
Sol. Let length of the flag be $x \mathrm{~cm}$ and breadth of the flag be $y \mathrm{~cm}$
or, $\quad \begin{aligned} 2(x+y) & =50 \\ x+y & =25\end{aligned}$

$$
\begin{equation*}
(x-3)(y+2)=x y-6 \tag{i}
\end{equation*}
$$

or, $\quad x y+2 x-3 y-6=x y-6$
or, $\quad 2 x-3 y=0$
On solving the eqns. (i) and (ii),

$$
\begin{equation*}
x=15 \mathrm{~cm} \text { and } y=10 \mathrm{~cm} \tag{ii}
\end{equation*}
$$

$\therefore$ Length of the flag $=15 \mathrm{~cm}$
and Breadth of the flag $=10 \mathrm{~cm}$

## Long Answer Type Questions

## 5 marks each

Q.1. A motorboat covers a distance of 16 km upstream and 24 km downstream in 6 hours. In the same time it covers a distance of 12 km upstream and 36 km downstream. Find the speed of the boat in still water and that of the stream.

$$
\mathrm{A}+\mathrm{C}[\mathrm{CBSE} \text { SQP, 2020-21] }
$$

Sol. Let speed of the boat in still water $=x \mathrm{~km} / \mathrm{hr}$, and speed of the current $=y \mathrm{~km} / \mathrm{hr}$ Downstream speed $=(x+y) \mathrm{km} / \mathrm{hr}$ Upstream speed $=(x-y) \mathrm{km} / \mathrm{hr}$

$$
\begin{equation*}
\frac{24}{x+y}+\frac{16}{x-y}=6 \tag{i}
\end{equation*}
$$

$$
\begin{equation*}
\frac{36}{x+y}+\frac{12}{x-y}=6 \tag{ii}
\end{equation*}
$$

Let

$$
\frac{1}{x+y}=u
$$

and $\quad \frac{1}{x-y}=v$
Put in the above equation, we get
or, $\quad 12 u+8 v=3$
or, $\quad 6 u+2 v=1$
Multiplying (iv) by 4 , we get

$$
\begin{equation*}
24 u+8 v=4 \tag{v}
\end{equation*}
$$

Subtracting (iii) by (v), we get

$$
\begin{array}{rlrl} 
& & 12 u & =1 \\
\Rightarrow & u & =1 / 12
\end{array}
$$

Putting the value of $u$ in (iv), we get, $v=1 / 4 \quad 1 / 2$
$\Rightarrow \quad \frac{1}{x+y}=\frac{1}{12}$
and $\quad \frac{1}{x-y}=\frac{1}{4}$
$\Rightarrow \quad x+y=12$ and $x-y=4$
Thus, speed of the boat in still water $=8 \mathrm{~km} / \mathrm{hr}, \quad 1 / 2$ Speed of the current $=4 \mathrm{~km} / \mathrm{hr}$.
$1 / 2$
(AI) Q. 2. It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for four hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. How long would it take for each pipe to fill the pool separately?
Sol. Let time taken to fill the pool by the larger diameter pipe $=x$ hr
and time taken to fill the pool by the smaller diameter pipe $=y \mathrm{hr}$

According to question,

$$
\begin{equation*}
\frac{1}{x}+\frac{1}{y}=\frac{1}{12} \tag{i}
\end{equation*}
$$

and $\quad \frac{4}{x}+\frac{9}{y}=\frac{1}{2}$
Let $\frac{1}{x}=a$ and $\frac{1}{y}=b$
Put in the above equations, we get

$$
\begin{align*}
a+b & =\frac{1}{12}  \tag{iii}\\
4 a+9 b & =\frac{1}{2} \tag{iv}
\end{align*}
$$

Multiplied by 4 to equ (iii)

$$
\begin{align*}
& 4 a+4 b=\frac{1}{3}  \tag{v}\\
& 4 a+9 b=\frac{1}{2}  \tag{vi}\\
& \frac{(-) \quad(-)}{-5 b}=\frac{(-)}{3}-\frac{1}{2} \\
&-5 b=\frac{2-3}{6} \\
& \therefore=\frac{1}{30}(\text { substitute in equ. (iii)) } \\
& \therefore \quad \ldots(\text { vi) } 1 / 2 \\
& a+\frac{1}{30}=\frac{1}{12}  \tag{1}\\
& a=\frac{1}{12}-\frac{1}{30} \\
& a=\frac{5-2}{60}=\frac{3}{60} \\
& a=\frac{1}{20} \\
& x=\frac{1}{a}=20 \\
& y=\frac{1}{b}=30 \tag{1}
\end{align*}
$$

Hence, time taken to fill the pool by the larger and smaller diameter pipe respectively 20 hrs and 30 hrs.
[AI) Q. 3. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

A [CBSE Delhi Set-I, II, III, 2019]
Sol. Let the speed of the boat in still water be $x \mathrm{~km} / \mathrm{hr}$ and speed of the stream be $y \mathrm{~km} / \mathrm{hr}$.
Given $\frac{30}{x-y}+\frac{44}{x+y}=10$
and $\frac{40}{x-y}+\frac{55}{x+y}=13$
...(ii) $11 / 2$

Solving (i) and (ii) to get

$$
\text { and } \quad \begin{array}{rlrl}
x+y & =11 \\
& x-y & =5 \tag{iv}
\end{array}
$$

Solving (iii) and (iv) to get $x=8, y=3$.
1
Speed of boat $=8 \mathrm{~km} / \mathrm{hr} \&$ speed of stream

$$
=3 \mathrm{~km} / \mathrm{hr} .1
$$

[CBSE Marking Scheme, 2019]

## Detailed Solution:

Let the speed of boat in still water $=x \mathrm{~km} / \mathrm{h}$
and the speed of current (stream) $=y \mathrm{~km} / \mathrm{h}$
Relative speed of boat in down stream $=x+y$
Relative speed of boat in up stream $=x-y \quad 1 / 2$ According to the first condition,

$$
\begin{equation*}
\therefore \quad \frac{30}{x-y}+\frac{44}{x+y}=10 \tag{i}
\end{equation*}
$$

According to the second condition,

$$
\begin{equation*}
\frac{40}{x-y}+\frac{55}{x+y}=13 \tag{ii}
\end{equation*}
$$

Let $\frac{1}{x-y}=u$ and $\frac{1}{x+y}=v$
So, $\quad 30 u+44 v=10$
...(iii) $1 / 2$

$$
\begin{equation*}
40 u+55 v=13 \tag{iv}
\end{equation*}
$$

From eq. (iii),

$$
\begin{align*}
30 u & =10-44 v \\
u & =\frac{10-44 v}{30} \tag{v}
\end{align*}
$$

Putting value of $u$ in eq. (iv), we get

$$
\left.\begin{array}{rlrl} 
& & 40\left(\frac{10-44 v}{3}\right)+55 v & =13 \\
& \Rightarrow & 4\left(\frac{10-44 v}{3}\right)+55 v & =13 \\
& \Rightarrow & 4(10-44 v)+165 v & =39 \\
& \Rightarrow & 40-176 v+165 v & =39 \\
\Rightarrow & & -11 v & =-1 \\
& \Rightarrow & & v
\end{array}\right)=\frac{1}{11} .
$$

Putting value of $v$ in eq. (v), we get

$$
\begin{array}{ll}
\Rightarrow & u=\frac{10-44 \times \frac{1}{11}}{30} \\
\Rightarrow & u=\frac{1}{5} \\
\text { So, } & u=\frac{1}{5} \text { and } v=\frac{1}{11}
\end{array}
$$

Now,

$$
\begin{array}{rlrl}
\Rightarrow & u & =\frac{1}{x-y}=\frac{1}{5} \\
& &  \tag{vi}\\
\text { and } & x-y & =5 \\
v & =\frac{1}{x+y}=\frac{1}{11}
\end{array}
$$

$\Rightarrow \quad x+y=11$
on adding eqs. (vi) and (vii), we get

$$
\begin{align*}
& & 2 x & =16 \\
\Rightarrow & & x & =8
\end{align*}
$$

Substitute value of $x$ in eq (vi), we get

$$
\begin{align*}
8-y & =5 \\
y & =3
\end{align*}
$$

Hence speed of boat in still water $=8 \mathrm{~km} / \mathrm{h}$ and speed of stream $=3 \mathrm{~km} / \mathrm{h}$
Q.4. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3 , determine the number.

C [CBSE SQP, 2017]
Sol. Let the ten's and unit digit be $y$ and $x$ respectively.
So, the number is $10 y+x$.
The number, when its digits are reversed, becomes $10 x+y$.
So, $\quad 7(10 y+x)=4(10 x+y)$
$\Rightarrow \quad 70 y+7 x=40 x+4 y$
$\Rightarrow \quad 70 y-4 y=40 x-7 x$
$\Rightarrow \quad 2 y=x \quad$...(i) 1
and $\quad x-y=3$
From (i) and (ii), we get

$$
\begin{equation*}
y=3 \text { and } x=6 \tag{ii}
\end{equation*}
$$

Hence, the number is 36 .
Q. 5.4 chairs and 3 tables cost $₹ 2100$ and 5 chairs and 2 tables cost ₹ 1750 . Find the cost of one chair and one table separately. A [Board Term-1, 2015]
Sol. Let cost of 1 chair be $₹ x$ and cost of 1 table be ₹ $y$
According to the question,

$$
\begin{equation*}
4 x+3 y=2100 \tag{i}
\end{equation*}
$$

and $\quad 5 x+2 y=1750$
Multiplying eqn. (i) by 2 and eqn. (ii) by 3, we get

$$
\begin{array}{r}
8 x+6 y=4200  \tag{iii}\\
15 x+6 y=5250
\end{array}
$$

...(iv) 1
eqn. (iv) - eqn. (iii),

$$
\begin{align*}
\Rightarrow & 7 x & =1050 \\
\therefore & x & =150 \tag{1}
\end{align*}
$$

Substituting the value of $x$ in (i) we get $y=500$
Thus, the cost of one chair and one table are ₹ 150 and ₹ 500 respectively.
[CBSE Marking Scheme, 2015] 2
(AI) Q. 6. Solve the following pair of equations:

$$
\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2 \text { and } \frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1
$$

U [Board Term-1, 2015]
Sol. Substituting $\quad \frac{1}{\sqrt{x}}=X$ and $\frac{1}{\sqrt{y}}=Y$

$$
\begin{equation*}
2 X+3 Y=2 \tag{i}
\end{equation*}
$$

and $\quad 4 X-9 Y=-1$
Multiplying eqn. (i) by 3 , and add in (ii), we get

$$
\begin{gather*}
4 X-9 Y=-1 \\
\frac{6 X+9 Y}{}=6 \\
10 X=5 \\
10 X=5 \text { or } X=\frac{5}{10} \Rightarrow \frac{1}{2}  \tag{2}\\
\frac{1}{\sqrt{x}}=\frac{1}{2} \quad \therefore x=4
\end{gather*}
$$

Putting the value of $X$ in eqn. (i), we get

$$
\begin{align*}
2 \cdot \frac{1}{2}+3 Y & =2 \\
3 Y & =2-1 \\
Y & =\frac{1}{3}  \tag{1}\\
\Rightarrow \quad Y & =\frac{1}{3} \text { or } \frac{1}{\sqrt{y}}=\frac{1}{3} \Rightarrow y=9
\end{align*}
$$

Hence, $x=4$ and $y=9$.

## Visual Case Based Questions

## 4 marks each

Note: Attempt any four sub parts from each question. Each sub part carries 1 mark
Q. 1. A test consists of 'True' or 'False' questions. One mark is awarded for every correct answer while $1 / 4$ mark is deducted for every wrong answer. A student knew answers to some of the questions. Rest of the questions he attempted by guessing. He answered 120 questions and got 90 marks.
[CBSE QB, 2021]

| Type of <br> Question | Marks given for <br> correct answer | Marks deducted <br> for wrong answer |
| :---: | :---: | :---: |
| True/False | 1 | 0.25 |

Let the no of questions whose answer is known to the student $x$ and questions attempted by cheating be $y$

$$
x+y=120
$$

$$
\frac{x-1}{4 y}=90
$$

Solving these two
$x=96$ and $y=24$
(i) If answer to all questions he attempted by guessing were wrong, then how many questions did he answer correctly?
Sol. He answered 96 questions correctly.
(ii) How many questions did he guess?

Sol. He attempted 24 questions by guessing.
(iii) If answer to all questions he attempted by guessing were wrong and answered 80 correctly, then how many marks he got ?
Sol. Marks $=80-1 / 4$ of $40=70$
(iv) If answer to all questions he attempted by guessing were wrong, then how many questions answered correctly to score 95 marks?

Sol. $x-1 / 4$ of $(120-x)=95$

$$
5 x=500, x=100
$$

Q. 2. Amit is planning to buy a house and the layout is given below. The design and the measurement has been made such that areas of two bedrooms and kitchen together is 95 sq.m.
[CBSE QB, 2021]


Based on the above information, answer the following questions:
(i) Form the pair of linear equations in two variables from this situation.
Sol. Area of two bedrooms $=10 x \mathrm{sq} . \mathrm{m}$

$$
\text { Area of kitchen }=5 y \text { sq. } \mathrm{m}
$$

$$
10 x+5 y=95
$$

$$
2 x+y=19
$$

Also,

$$
\begin{array}{r}
x+2+y=15 \\
x+y=13
\end{array}
$$

(ii) Find the length of the outer boundary of the layout.

Sol. Length of outer boundary $=12+15+12+15$

$$
=54 \mathrm{~m}
$$

(iii) Find the area of each bedroom and kitchen in the layout.
Sol. On solving two equation part (i)
$x=6 \mathrm{~m}$ and $y=7 \mathrm{~m}$
area of bedroom $=5 \times 6=30 \mathrm{~m}$
area of kitchen $=5 \times 7=35 \mathrm{~m}$
(iv) Find the area of living room in the layout.

Sol. Area of living room $=(15 \times 7)-30$

$$
\begin{aligned}
& =105-30 \\
& =75 \mathrm{sq} . \mathrm{m}
\end{aligned}
$$

(v) Find the cost of laying tiles in kitchen at the rate of ₹ 50 per sq. m
Sol. Total cost of laying tiles in the kitchen $=₹ 50 \times 35$

$$
\text { = ₹ } 1750
$$

Q. 3. It is common that Governments revise travel fares from time to time based on various factors such as inflation (a general increase in prices and fall in the purchasing value of money) on different types of vehicles like auto, Rickshaws, taxis, Radio cab etc. The auto charges in a city comprise of a fixed charge together with the charge for the distance covered. Study the following situations
[CBSE QB, 2021]



| Name of the city | Distance travelled <br> $(\mathrm{km})$ | Amount paid <br> $(₹)$ |
| :---: | :---: | :---: |
| City A | 10 | 75 |
|  | 15 | 110 |
| City B | 8 | 9 |
|  | 14 | 145 |

Situation 1: In city A, for a journey of 10 km , the charge paid is ₹ 75 and for a journey of 15 km , the charge paid is ₹ 110 .
Situation 2: In a city B, for a journey of 8 km , the charge paid is ₹ 91 and for a journey of 14 km , the charge paid is ₹ 145 .

## Refer situation 1

(i) If the fixed charges of auto rickshaw be $₹ x$ and the running charges be $₹ y \mathrm{~km} / \mathrm{hr}$, the pair of linear equations representing the situation is
(a) $x+10 y=110, x+15 y=75$
(b) $x+10 y=75, x+15 y=110$
(c) $10 x+y=110,15 x+y=75$
(d) $10 x+y=75,15 x+y=110$

Sol. Correct option: (b).
Explanation: According to given situation, we have

$$
\begin{align*}
& x+10 y=75  \tag{i}\\
& x+15 y=110 \tag{ii}
\end{align*}
$$

(ii) A person travels a distance of 50 km . The amount he has to pay is
(a) ₹ 155
(b) ₹ 255
(c) ₹ 355
(d) ₹ 455

Sol. Correct option: (c).
Explanation: Solving two equations,

$$
\begin{aligned}
& x+10 y=75 \\
& x+15 y=110 \\
&-\quad-\quad- \\
& \hline-5 y=-35 \\
& y=7
\end{aligned}
$$

Now, putting $y=7$ in equation (i)

$$
\begin{aligned}
x+10 \times 7 & =75 \\
x+70 & =75 \\
x & =75-70 \\
x & =5
\end{aligned}
$$

Now, if a person travels a distance of 50 km
then, $\quad$ amount $=x+50 y$

$$
\begin{aligned}
& =5+50 \times 7 \\
& =5+350 \\
& =355
\end{aligned}
$$

Refer situation 2
(iii) What will a person have to pay for travelling a distance of 30 km ?
(a) ₹ 185
(b) ₹ 289
(c) ₹ 275
(d) ₹ 305

Sol. Correct option: (b).
(iv) The graph of lines representing the conditions are: (situation 2)
(a)

(b)

(c)

(d)


Sol. Correct option: (c).
(AI) Q. 4. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in $\mathbf{1}$ hour. $C+A E$

(i) Assuming that the speed of first car and second car be $u \mathrm{~km} / \mathrm{h}$ and $v \mathrm{~km} / \mathrm{h}$ respectively.
What is the relative speed of both cars while they are travelling in the same direction?
(a) $(u+v) \mathrm{km} / \mathrm{hr}$
(b) $(u-v) \mathrm{km} / \mathrm{hr}$
(c) $(u / v) \mathrm{km} / \mathrm{hr}$
(d) $(u v) \mathrm{km} / \mathrm{hr}$

Sol. Correct Option: (b)
Explanation: Relative speed of both cars while they are travelling in same direction $=(u-v) \mathrm{km} / \mathrm{hr}$.
(ii) What is the relative speed of both cars while they are travelling towards each other?
(a) $(u+v) \mathrm{km} / \mathrm{hr}$
(b) $(u-v) \mathrm{km} / \mathrm{hr}$
(c) $(u / v) \mathrm{km} / \mathrm{hr}$
(d) $(u v) \mathrm{km} / \mathrm{hr}$

Sol. Correct Option: (a)
Explanation: Relative speed of both cars while they are travelling in opposite directions i.e., travelling towards each other $=(u+v) \mathrm{km} / \mathrm{hr}$.
(iii) What is the actual speed of one car?
(a) $60 \mathrm{~km} / \mathrm{hr}$
(b) $40 \mathrm{~km} / \mathrm{hr}$
(c) $100 \mathrm{~km} / \mathrm{hr}$
(d) $20 \mathrm{~km} / \mathrm{hr}$

Sol. Correct Option: (a)
Explanation: Let the speeds of first car and second car be $u \mathrm{~km} / \mathrm{hr}$ and $v \mathrm{~km} / \mathrm{hr}$ respectively.
According to the given information.

$$
\begin{align*}
\text { i.e,. } & 5(u-v) & =100 \\
& u-v & =20 \tag{i}
\end{align*}
$$

$$
\begin{equation*}
\text { and } \quad u+v=100 \tag{ii}
\end{equation*}
$$

Solving eqs. (i) and (ii), we get $u=60 \mathrm{~km} / \mathrm{hr}$.
(iv) What is the actual speed of other car?
(a) $60 \mathrm{~km} / \mathrm{hr}$
(b) $40 \mathrm{~km} / \mathrm{hr}$
(c) $100 \mathrm{~km} / \mathrm{hr}$
(d) $20 \mathrm{~km} / \mathrm{hr}$

Sol. Correct Option: (b)
Explanation: From above question 3, refering to the solution of both equations

$$
v=40 \mathrm{~km} / \mathrm{hr} .
$$

(v) The given problem is based on which mathematical concept
(a) Pair of linear equations
(b) Quadratic equations
(c) Polynomials
(d) none of the above

Sol. Correct Option: (a)
Explanation: The given problem is based on pair of linear equations.
Q. 5. John and Jivanti are playing with the marbles in the playground. They together have 45 marbles and John has 15 marbles more than Jivanti.

(i) The number of marbles Jivanti had:
(a) 15
(b) 30
(c) 40
(d) 5

Sol. Correct Option: (a)
Explanation: Let the no. of marbles, John and Jivanti have, be $x$ and $y$ respectively.
According to the given information,

$$
\text { and } \quad \begin{align*}
x+y & =45  \tag{i}\\
& x-y
\end{align*}
$$

Solving eqs. (i) and (ii), we get

$$
x=30 \text { and } y=15
$$

(ii) The number of marbles John had:
(a) 40
(b) 30
(c) 15
(d) 20

Sol. Correct Option: (b)
Explanation: According to the solution of question 1 , we get $\quad x=30$.
(iii) If 45 is replaced by 55 in the above case discussed in the question, then the number of marbles Jivanti have:
(a) 15
(b) 30
(c) 20
(d) 35

Sol. Correct Option: (c)
Explanation: According to given problem,

$$
\begin{align*}
& x+y  \tag{i}\\
\text { and } & =55  \tag{ii}\\
x-y & =15
\end{align*}
$$

Solving eqs. (i) and (ii), we get

$$
x=35 \text { and } y=20 .
$$

(iv) According to the question 3, the number of marbles John have:
(a) 30
(b) 40
(c) 45
(d) 35

Sol. Correct Option: (d)
Explanation: From above question 3, we get $x=35$. Hence, John had 35 marbles.
(v) The given problem is based on which mathematical concept?
(a) pair of linear equations
(b) Quadratic equations
(c) Polynomials
(d) None of the above

Sol. Correct Option: (a)
Explanation: The given problem is based on pair of linear equations.

